

Quark-Gluon Antenna Functions from Neutralino Decay

A. Gehrmann–De Ridder^a, T. Gehrmann^b and E.W.N. Glover^c

^a *Institute for Theoretical Physics, ETH, CH-8093 Zürich, Switzerland*

^b *Institut für Theoretische Physik, Universität Zürich, Winterthurerstrasse 190,
CH-8057 Zürich, Switzerland*

^c *Institute for Particle Physics Phenomenology, University of Durham, South Road,
Durham DH1 3LE, England*

Abstract

The computation of exclusive QCD jet observables at higher orders requires a method for the subtraction of infrared singular configurations arising from multiple radiation of real partons. One commonly used method at next-to-leading order (NLO) is based on the antenna factorization of colour-ordered matrix elements, and uses antenna functions to subtract the real radiation singularities. Up to now, NLO antenna functions could be derived in a systematic manner only for hard quark-antiquark pairs, while the gluon-gluon and quark-gluon antenna functions were constructed from their limiting behaviour. In this paper, we show that antenna functions for hard quark-gluon pairs can be systematically derived from an effective Lagrangian describing heavy neutralino decay. The infrared structure of the colour-ordered neutralino decay matrix elements at NLO and NNLO is shown to agree with the structure observed for parton radiation off a quark-gluon antenna.

1 Introduction

Experimental measurements of jet production observables are among the most sensitive tests of the theory of Quantum Chromodynamics (QCD), and yield very accurate determinations of QCD parameters [1], especially of the strong coupling constant α_s . At present, the precision of many of these determinations is limited not by the quality of the experimental data, but by the error on the theoretical (next-to-leading order, NLO) calculations used for the extraction of the QCD parameters. To improve upon this situation, an extension of the theoretical calculations to next-to-next-to-leading order (NNLO) is therefore mandatory.

In the recent past, many ingredients to NNLO calculations of collider observables have been derived, including the universal three-loop QCD splitting functions [2] which governs the evolution of parton distribution functions at NNLO. The massless two-loop $2 \rightarrow 2$ and $1 \rightarrow 3$ matrix elements relevant to NNLO jet production have been computed [3] using several innovative methods [4], and are now available for all processes of phenomenological relevance. The one-loop corrections to $2 \rightarrow 3$ and $1 \rightarrow 4$ matrix elements have been known for longer and form part of NLO calculations of the respective multi-jet observables [5, 6]. These NLO matrix elements naturally contribute to NNLO jet observables of lower multiplicity if one of the partons involved becomes soft or collinear [7]. In these cases, the infrared singular parts of the matrix elements need to be extracted and integrated over the phase space appropriate to the unresolved configuration to make the infrared pole structure explicit. Methods for the extraction of soft and collinear limits of one-loop matrix elements are worked out in detail in the literature [8]. As a final ingredient, contributions from the tree level $2 \rightarrow 4$ and $1 \rightarrow 5$ processes also contribute to $(2 \rightarrow 2)$ - and $(1 \rightarrow 3)$ -type jet observables at NNLO. These contain double real radiation singularities corresponding to two partons becoming simultaneously soft and/or collinear [9, 10]. To determine the contribution to NNLO jet observables from these configurations, one has to find two-parton subtraction terms which coincide with the full matrix element and are still sufficiently simple to be integrated analytically in order to cancel their infrared pole structure with the two-loop virtual and the one-loop single-unresolved contributions. Several methods have been proposed recently to accomplish this task [11]. Up to now, only one method has been fully worked through for an observable of physical interest: using the sector decomposition algorithm [12, 13] to analytically decompose both phase space and loop integrals into their Laurent expansion in dimensional regularization, and subsequent numerical computation of the coefficients of this expansion, results were obtained for $e^+e^- \rightarrow 2j$ [14] and $pp \rightarrow H + X$ [15] at NNLO. In contrast to all other approaches, in the sector decomposition method one does not have to integrate the subtraction term analytically.

In [16], we described the construction of NNLO subtraction terms for $e^+e^- \rightarrow 2j$ based on full four-parton tree-level and three-parton one-loop matrix elements, which can be integrated analytically over the appropriate phase spaces [13]. Subtraction terms derived from full matrix elements can be viewed as antenna functions, encapsulating all singular limits due to unresolved partonic emission between two colour-connected hard partons [6, 17]. In particular, process-independent antenna functions describing arbitrary QCD multi-particle processes can be directly related to three-parton matrix elements at NLO (one unresolved parton radiating between two colour-connected hard partons) and four-parton matrix elements at NNLO (two unresolved partons radiating between two colour-connected hard partons).

Up to now, antenna subtraction terms (at NLO) were obtained by construction (i.e. by inspecting all limits they had to contain), in part from the full matrix elements, and in part by using supersymmetric (SUSY) relations between matrix elements containing fermions and bosons [6]. A systematic procedure to derive antenna functions at NLO and beyond is not available up to now: this paper aims to contribute to such a formalism by showing that quark-gluon antenna functions can be derived systematically from physical matrix elements obtained from an effective Lagrangian.

The NNLO subtraction terms derived from four-parton matrix elements with a hard quark-antiquark pair in [16] were used subsequently [18] to compute the $\alpha_s^3 C_F^3$ -correction to $e^+e^- \rightarrow 3j$ at NNLO. To extend this calculation to the remaining colour factors, further subtraction terms must be derived. In particular, the subtraction terms of [16] are sufficient only for processes where unresolved partons are radiated from hard quark-antiquark pairs: they form the quark-antiquark antenna functions at NNLO. Besides quark-antiquark antennae, $e^+e^- \rightarrow 3j$ also contains radiation from hard quark-gluon pairs, the quark-gluon antenna function. In the spirit of [16], it should be possible to extract these antenna functions from the matrix elements appearing in the NLO and NNLO corrections to a physical one-particle decay process yielding a quark-gluon

final state at leading order. It is the purpose of this letter to show that such a process can be described by an appropriate colour ordering of the decay of a massive neutralino into a massless gluino and a gluon, and to derive the resulting quark-gluon antenna subtraction terms at NLO and NNLO.

2 Effective Lagrangian and Feynman rules

To obtain the correct quantum numbers for a quark-gluon antenna function, one has to consider the decay of an off-shell spin-1/2 particle into an on-shell spin-1/2 particle (massless quark) and an on-shell spin-1 particle (gluon). Since the final state quark is in the triplet representation of $SU(3)$, while the gluon is in the octet representation, this implies that the initial state spin-1/2 off-shell particle should also be in the triplet representation. $SU(3)$ gauge invariance does however forbid external off-shell states.

In the colour-ordered formulation of QCD tree-level amplitudes [19,20], one decouples the colour quantum numbers of the partons from their Lorentz and Dirac structure. Using this formulation, one can in particular represent a parton in the adjoint representation as superposition of two partons (with identical momenta) in the fundamental representation. It is thus possible to construct the colour ordered quark-gluon antenna functions from the $SU(3)$ gauge-invariant decay of an off-shell spin-1/2 *singlet* state (neutralino) into a spin-1/2 octet state (massless gluino) and a spin-1 octet state (gluon), as we shall show below.

This decay process occurs in the minimal supersymmetric standard model (MSSM, [21]), where it is mediated through a loop involving supersymmetric particles. For the purpose of this study, it is sufficient to describe this process through an effective Lagrangian, whose parameters are obtained by integrating out the virtual particles in the loop. In the context of the electroweak sector of the MSSM, this effective Lagrangian was first derived by Haber and Wyler [22], to describe heavy neutralino decay into a light neutralino and a photon. Its generalization to neutralino decay into gluino and gluon is straightforward:

$$\mathcal{L}_{\text{int}} = i\eta\bar{\psi}_{\tilde{g}}^a\sigma^{\mu\nu}\psi_{\tilde{\chi}}F_{\mu\nu}^a + (\text{h.c.}) , \quad (2.1)$$

which couples a gluino ($\bar{\psi}_{\tilde{g}}^a$) and a neutralino ($\psi_{\tilde{\chi}}$) to the QCD field strength tensor $F_{\mu\nu}^a$. The coupling η has inverse mass dimension, and the commutator of the γ -matrices is

$$\sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu] .$$

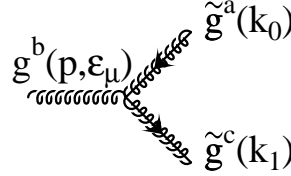
It should be noted that this process was discussed previously in the literature in [23], where however no effective Lagrangian was stated.

The Feynman rules following from the Lagrangian (2.1) are

$$\begin{aligned} \text{Top Diagram: } \tilde{\chi}(p) \rightarrow \tilde{g}^a(k_0) + g^b(k_1, \epsilon_{1,\mu}) &= -i\eta\delta^{ab}\sigma_{\mu\nu}k_1^\nu , \\ \text{Bottom Diagram: } \tilde{\chi}(p) \rightarrow \tilde{g}^a(k_0) + g^b(k_1, \epsilon_{1,\mu}) + g^c(k_2, \epsilon_{2,\nu}) &= -g_s\eta f^{abc}\sigma_{\mu\nu} , \end{aligned} \quad (2.2)$$

where the arrow indicates the direction of fermion flow. It should be noted that Majorana particles also have a fermion flow direction, which does however not coincide with the fermion number flow. The momenta are always incoming.

Besides these Feynman rules and the standard QCD Feynman rules, one needs moreover the Feynman rule for the gluon-gluino-gluino coupling [24]



$$= -g_s f^{abc} \gamma^\mu . \quad (2.3)$$

The effective coupling η can be computed in the MSSM, it was discussed in [22, 23]. In the present context, its value is irrelevant, but we do have to take into account that η is renormalized at one loop. Its QCD renormalization constant reads

$$Z_\eta = 1 - \frac{\alpha_s}{2\pi} \frac{1}{\epsilon} \left(\frac{\beta_0}{2} + \frac{3N}{4} \right) + \mathcal{O}(\alpha_s^2) , \quad (2.4)$$

with

$$\beta_0 = \frac{11N - 2N_F}{6} . \quad (2.5)$$

In SUSY QCD, both Z_η and β_0 are modified. Throughout this paper, we systematically ignore the SUSY QCD corrections and restrict ourselves to the subclass of contributions which preserve the QCD renormalizations (2.4) and (2.5).

3 Colour-ordered amplitudes in neutralino decay

The basic process for the decay of a neutralino into a gluino plus partons is $\tilde{\chi}(q) \rightarrow \tilde{g}(p_1)g(p_3)$. Its amplitude reads

$$i\eta \delta^{a_1 a_3} M_{gg}^0(p_1, p_3) .$$

To display the colour-ordered structure of this amplitude, and to illustrate the relation to quark-gluon amplitudes, we multiply it with $\sqrt{2}T_{i_1 i_2}^{a_1}$ [20]. In the squared amplitude, this factor corresponds to inserting unity since

$$\sqrt{2}T_{i_1 i_2}^{a_1} \sqrt{2}T_{i_2 i_1}^{a_1'} = \delta^{a_1 a_1'} .$$

The resulting amplitude is then

$$\mathcal{M}_{g_1 g_3}^0 = i\eta \sqrt{2} T_{i_1 i_2}^{a_3} M_{gg}^0(p_1, p_3) . \quad (3.1)$$

From this structure, it can be seen that the amplitude contains two colour connected (hard) partons and therefore two antennae:

1. A quark-gluon antenna, with quark momentum p_1 , quark colour index i_1 , gluon momentum p_3 and gluon colour index a_3 .
2. An antiquark-gluon antenna, with antiquark momentum p_1 , antiquark colour index i_2 , gluon momentum p_3 and gluon colour index a_3 .

This derivation is displayed pictorially in Figure 1. It becomes evident that the Majorana nature of the gluino allows it to represent both a quark and an anti-quark.

The squared matrix element is

$$\mathcal{T}_{gg}^0(q^2) \equiv |\mathcal{M}_{g_1 g_3}^0|^2 = \eta^2 (N^2 - 1) |M_{gg}^0(p_1, p_3)|^2 = 4 (N^2 - 1) \eta^2 (1 - \epsilon) (q^2)^2 . \quad (3.2)$$

$\mathcal{T}_{gg}^0(q^2)$ serves as normalization for antenna functions obtained from higher order corrections to this matrix element.

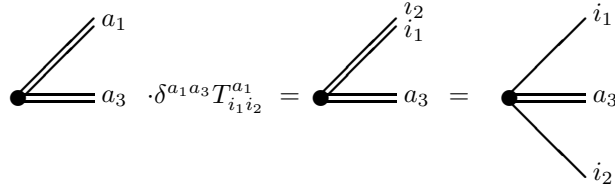


Figure 1: Colour flow contained in tree level decay $\tilde{\chi} \rightarrow \tilde{g}g$. Double (single) lines denote adjoint (fundamental) colour indices.

To demonstrate the cancellation of infrared divergences at NLO, we compute the renormalized one-loop QCD correction to the $\tilde{\chi}(q) \rightarrow \tilde{g}(p_1)g(p_3)$ decay,

$$\begin{aligned} \mathcal{T}_{\tilde{g}g}^1(q^2) &\equiv 2\text{Re}|\mathcal{M}_{\tilde{g}_1 g_3}^0 \mathcal{M}_{\tilde{g}_1 g_3}^{1,*}| \\ &= \left(\frac{\alpha_s}{2\pi}\right) 2(q^2)^{-\epsilon} \mathcal{T}_{\tilde{g}g}^0(q^2) \left\{ N \left[-\frac{1}{\epsilon^2} - \frac{5}{3\epsilon} + \frac{7\pi^2}{12} + \left(-1 + \frac{7}{3}\zeta_3\right)\epsilon + \left(-3 - \frac{73\pi^4}{1440}\right)\epsilon^2 \right] \right. \\ &\quad \left. + \frac{N_F}{6\epsilon} + \mathcal{O}(\epsilon^3) \right\}. \end{aligned} \quad (3.3)$$

The infrared poles of this one-loop correction can be expressed in terms of the infrared singularity operator [25]

$$\mathbf{I}_{q\bar{q}}^{(1)}(\epsilon, q^2) = -\frac{e^{\epsilon\gamma}}{2\Gamma(1-\epsilon)} \left[N \left(\frac{1}{\epsilon^2} + \frac{3}{4\epsilon} + \frac{\beta_0}{2N\epsilon} \right) (-q^2)^{-\epsilon} \right] \quad (3.4)$$

as

$$\mathcal{Poles}(\mathcal{T}_{\tilde{g}g}^1(q^2)) = \left(\frac{\alpha_s}{2\pi}\right) 4\text{Re}\mathbf{I}_{q\bar{q}}^{(1)}(\epsilon, q^2) \mathcal{T}_{\tilde{g}g}^0(q^2). \quad (3.5)$$

This expression has to be compared to the $2\text{Re}\mathbf{I}_{q\bar{q}}^{(1)}(\epsilon, q^2)$, which is obtained in the decay of a virtual photon into a quark-antiquark pair $\gamma^* \rightarrow q\bar{q}$ at one loop [16]. The factor 4 in (3.5) appears since the leading order process $\tilde{\chi} \rightarrow \tilde{g}g$ contains two distinct quark-gluon antennae, in contrast to the single quark-antiquark antenna contained in $\gamma^* \rightarrow q\bar{q}$.

4 NLO antenna functions

Two different emissions off a quark-gluon pair appear at NLO: either the emission of an additional gluon or the splitting of the gluon into a quark-antiquark pair. In the context of the neutralino decay, these correspond to the tree level processes $\tilde{\chi} \rightarrow \tilde{g}gg$ and $\tilde{\chi} \rightarrow \tilde{g}q\bar{q}$.

The tree level amplitude for $\tilde{\chi}(q) \rightarrow \tilde{g}(p_1)g(p_3)g(p_4)$ contains only a single colour structure, $f^{a_1 a_3 a_4}$. In order to relate this colour structure to the colour-ordered quark-gluon antennae, we multiply with $\sqrt{2}T_{i_1 i_2}^{a_1}$:

$$\mathcal{M}_{\tilde{g}_1 g_3 g_4}^0 = i\eta g(-i\sqrt{2}) \left[(T^{a_3} T^{a_4})_{i_1 i_2} - (T^{a_4} T^{a_3})_{i_1 i_2} \right] M_{\tilde{g}gg}^0(p_1, p_3, p_4), \quad (4.1)$$

showing that the two colour-ordered amplitudes in this matrix element (corresponding to the two different orderings of the gluons along the quark-antiquark $i_1 i_2$ colour line) are equivalent to each other up to an overall sign because of the identical momenta of quark and antiquark. Squaring the matrix element and dividing by a symmetry factor to account for identical gluons in the final state yields

$$\frac{1}{2} |\mathcal{M}_{\tilde{g}_1 g_3 g_4}^0|^2 = \eta^2 g^2 (N^2 - 1) N \frac{1}{2} |M_{\tilde{g}gg}^0(p_1, p_3, p_4)|^2, \quad (4.2)$$

with

$$\begin{aligned} \frac{1}{2} |M_{ggg}^0(p_1, p_3, p_4)|^2 &= 4(1-\epsilon) \left(\frac{2s_{134}^2 s_{14}}{s_{13} s_{34}} + \frac{2s_{134}^2 s_{13}}{s_{14} s_{34}} + \frac{(1-\epsilon) s_{134} s_{34}}{s_{13}} + \frac{(1-\epsilon) s_{134} s_{34}}{s_{14}} \right. \\ &\quad \left. + \frac{2s_{13} s_{14}}{s_{34}} + 6s_{134} + (1-\epsilon)(s_{13} + s_{14}) \right) - 8s_{134}. \end{aligned} \quad (4.3)$$

The behaviour of this matrix element in the kinematical limits where one parton becomes unresolved is as follows:

1. Collinear limits:

$$\begin{aligned} \frac{1}{2} |\mathcal{M}_{\tilde{g}_1 g_3 g_4}^0|^2 &\xrightarrow{\tilde{g}_1 \parallel g_3} (4\pi\alpha_s) \mathcal{T}_{\tilde{g}g}^0(s_{134}) \frac{1}{s_{13}} N P_{q \rightarrow qg}(z), \\ \frac{1}{2} |\mathcal{M}_{\tilde{g}_1 g_3 g_4}^0|^2 &\xrightarrow{\tilde{g}_1 \parallel g_4} (4\pi\alpha_s) \mathcal{T}_{\tilde{g}g}^0(s_{134}) \frac{1}{s_{14}} N P_{q \rightarrow qg}(z), \\ \frac{1}{2} |\mathcal{M}_{\tilde{g}_1 g_3 g_4}^0|^2 &\xrightarrow{g_3 \parallel g_4} (4\pi\alpha_s) \mathcal{T}_{\tilde{g}g}^0(s_{134}) \frac{1}{s_{34}} N P_{g \rightarrow gg}(z), \end{aligned} \quad (4.4)$$

with z being the momentum fraction of one of the collinear partons and the splitting functions

$$P_{q \rightarrow qg}(z) = \frac{1+z^2}{1-z} - \epsilon(1-z), \quad P_{g \rightarrow gg}(z) = 2 \left[\frac{z}{1-z} + \frac{1-z}{z} + z(1-z) \right].$$

2. Soft limits:

$$\begin{aligned} \frac{1}{2} |\mathcal{M}_{\tilde{g}_1 g_3 g_4}^0|^2 &\xrightarrow{g_3 \rightarrow 0} (4\pi\alpha_s) \mathcal{T}_{\tilde{g}g}^0(s_{134}) N \frac{2s_{14}}{s_{13} s_{34}}, \\ \frac{1}{2} |\mathcal{M}_{\tilde{g}_1 g_3 g_4}^0|^2 &\xrightarrow{g_4 \rightarrow 0} (4\pi\alpha_s) \mathcal{T}_{\tilde{g}g}^0(s_{134}) N \frac{2s_{13}}{s_{14} s_{34}}. \end{aligned} \quad (4.5)$$

Comparing these limits to the limits of colour-ordered QCD matrix elements, one observes that the collinear $q \rightarrow qg$ limit contains a colour factor $N/2$ in QCD, while the collinear $\tilde{g} \rightarrow \tilde{g}g$ limit derived here contains a colour factor N . This is precisely what was expected from the discussion in the previous section, since the neutralino decay matrix element considered here contains both a quark-gluon and an antiquark-gluon antenna. On the other hand, the collinear $g \rightarrow gg$ limit appears here with the same colour factor as in colour ordered QCD matrix elements, indicating that the collinear $g \rightarrow gg$ limit is to be split between both antenna functions, as discussed in [6, 17]. Finally, the matrix element derived here contains two soft limits with the soft eikonal factors as expected in QCD, again reflecting the presence of two antennae.

Integration over the dipole phase space [13] yields

$$\begin{aligned} \mathcal{T}_{\tilde{g}gg}^1(q^2) &\equiv \int d\Phi_{D, \tilde{g}gg} \frac{1}{2} |\mathcal{M}_{\tilde{g}_1 g_3 g_4}^0|^2 \\ &= \left(\frac{\alpha_s}{2\pi} \right) N \mathcal{T}_{\tilde{g}g}^0(q^2) (q^2)^{-\epsilon} \left[\frac{2}{\epsilon^2} + \frac{10}{3\epsilon} + \frac{34}{3} - \frac{7\pi^2}{6} + \left(\frac{209}{6} - \frac{35\pi^2}{18} - \frac{50}{3}\zeta_3 \right) \epsilon \right. \\ &\quad \left. + \left(\frac{421}{4} - \frac{119\pi^2}{18} - \frac{250}{9}\zeta_3 - \frac{71\pi^4}{720} \right) \epsilon^2 + \mathcal{O}(\epsilon^3) \right]. \end{aligned} \quad (4.6)$$

The tree level amplitude for $\tilde{\chi}(q) \rightarrow \tilde{g}(p_1)q(p_3)\bar{q}(p_4)$ contains only a single colour structure $T_{i_3 i_4}^{a_1}$, which is again contracted with $\sqrt{2}T_{i_1 i_2}^{a_1}$

$$\mathcal{M}_{\tilde{g}_1 q_3 \bar{q}_4}^0 = i\eta g \frac{1}{\sqrt{2}} \left(\delta_{i_1 i_4} \delta_{i_3 i_2} - \frac{1}{N} \delta_{i_1 i_2} \delta_{i_3 i_4} \right) M_{\tilde{g}q\bar{q}}^0(p_1, p_3, p_4), \quad (4.7)$$

yielding

$$|\mathcal{M}_{\tilde{g}_1 q_3 \bar{q}_4}^0|^2 = \eta^2 g^2 \frac{N^2 - 1}{2} |M_{\tilde{g} q \bar{q}}^0(p_1, p_3, p_4)|^2, \quad (4.8)$$

with

$$|M_{\tilde{g} q \bar{q}}^0(p_1, p_3, p_4)|^2 = 4(1 - \epsilon) \left(2 \frac{(s_{13} + s_{14})^2}{s_{34}} + 2(s_{13} + s_{14}) \right) - 16 \frac{s_{13} s_{14}}{s_{34}}. \quad (4.9)$$

The only singular configuration contained in this matrix element is the collinear quark-antiquark limit, which is as follows:

$$|\mathcal{M}_{\tilde{g}_1 q_3 \bar{q}_4}^0|^2 \xrightarrow{q_3 \parallel \bar{q}_4} (4\pi\alpha_s) \mathcal{T}_{\tilde{g}g}^0(s_{134}) \frac{1}{s_{34}} P_{g \rightarrow q\bar{q}}(z), \quad (4.10)$$

with the collinear splitting function

$$P_{g \rightarrow q\bar{q}}(z) = 1 - \frac{2z(1-z)}{1-\epsilon}.$$

Integration over the dipole phase space [13] and summing over final state quark flavours yields

$$\begin{aligned} \mathcal{T}_{\tilde{g} q \bar{q}}^1(q^2) &\equiv \int d\Phi_{D, \tilde{g} q \bar{q}} \sum_q |\mathcal{M}_{\tilde{g}_1 q_3 \bar{q}_4}^0|^2 \\ &= \left(\frac{\alpha_s}{2\pi} \right) N_F \mathcal{T}_{\tilde{g}g}^0(q^2) (q^2)^{-\epsilon} \left[-\frac{1}{3\epsilon} - 1 + \left(-3 + \frac{7\pi^2}{36} \right) \epsilon + \left(-9 + \frac{7\pi^2}{12} + \frac{25}{9} \zeta_3 \right) \epsilon^2 + \mathcal{O}(\epsilon^3) \right]. \end{aligned} \quad (4.11)$$

Summing over both three parton final states, we find

$$\mathcal{Poles}(\mathcal{T}_{\tilde{g}gg}^1(q^2) + \mathcal{T}_{\tilde{g}q\bar{q}}^1(q^2)) = -\left(\frac{\alpha_s}{2\pi} \right) 4\text{Re}\mathcal{I}_{qg}^{(1)}(\epsilon, q^2) \mathcal{T}_{\tilde{g}g}^0(q^2), \quad (4.12)$$

such that the NLO corrected neutralino decay rate into gluino plus partons is finite:

$$\mathcal{Poles}(\mathcal{T}_{\tilde{g}g}^1(q^2)) + \mathcal{Poles}(\mathcal{T}_{\tilde{g}gg}^1(q^2) + \mathcal{T}_{\tilde{g}q\bar{q}}^1(q^2)) = 0. \quad (4.13)$$

It has to be emphasised in this context that we considered here only the QCD corrections to the neutralino decay, not the SUSY QCD corrections. At NLO, inclusion of SUSY QCD corrections would both modify the renormalization (2.4) of the effective coupling η , and include a real radiation contribution from the three gluino final state (which has however the same singularity structure as the $\tilde{g}q\bar{q}$ final state). The omission of these corrections is deliberate, since we want to derive the QCD quark-gluon antenna functions. In the following section, we demonstrate that the NNLO infrared singularity structure of QCD quark-gluon antenna functions is reproduced correctly by the QCD corrections to neutralino decay into gluino plus partons.

5 Structure of NNLO antenna functions

In the NNLO calculation of jet observables, two different types of antenna functions are required: (a) the one-loop correction to the three-parton antenna functions which appeared at NLO in tree-level form, and (b) the tree-level four-parton antenna functions. In this section, we present all neutralino decay matrix elements needed for the derivation of these antenna functions, and demonstrate that these matrix elements contain the same infrared singularities as processes involving final state emission off a quark-gluon antenna.

The renormalized one-loop corrections to the three-parton antenna functions have the same colour structure as their tree level counterparts listed above. In their computation, closed gluino loops are omitted, since these form part of the SUSY QCD corrections. Consequently, renormalization of the coupling constant is done using the QCD β -function. To expose the infrared structure of the resulting one-loop matrix elements,

they are integrated over the corresponding dipole phase space [13], yielding

$$\begin{aligned}
\mathcal{T}_{\tilde{g}gg}^2(q^2) &\equiv \int d\Phi_{D,\tilde{g}gg} \frac{1}{2} 2\text{Re} \left(\mathcal{M}_{\tilde{g}_1 g_3 g_4}^0 \mathcal{M}_{\tilde{g}_1 g_3 g_4}^{1,*} \right) \\
&= \left(\frac{\alpha_s}{2\pi} \right)^2 \mathcal{T}_{\tilde{g}g}^0(q^2) (q^2)^{-2\epsilon} \left[N^2 \left(-\frac{9}{2\epsilon^4} - \frac{56}{3\epsilon^3} + \frac{1}{\epsilon^2} \left(-\frac{1835}{36} + \frac{71\pi^2}{12} \right) \right. \right. \\
&\quad \left. \left. + \frac{1}{\epsilon} \left(-\frac{20977}{108} + \frac{209\pi^2}{12} + 72\zeta_3 \right) + \left(-\frac{19499}{27} + \frac{4195\pi^2}{72} + \frac{695}{3}\zeta_3 - \frac{995\pi^4}{720} \right) \right) \right. \\
&\quad \left. + NN_F \left(\frac{4}{3\epsilon^3} + \frac{20}{9\epsilon^2} + \frac{1}{\epsilon} \left(\frac{275}{36} - \frac{7\pi^2}{9} \right) + \left(\frac{287}{12} - \frac{35\pi^2}{27} - \frac{100}{9}\zeta_3 \right) \right) + \mathcal{O}(\epsilon) \right], \quad (5.1)
\end{aligned}$$

$$\begin{aligned}
\mathcal{T}_{\tilde{g}q\bar{q}}^2(q^2) &\equiv \int d\Phi_{D,\tilde{g}q\bar{q}} 2\text{Re} \left(\mathcal{M}_{\tilde{g}_1 q_3 \bar{q}_4}^0 \mathcal{M}_{\tilde{g}_1 q_3 \bar{q}_4}^{1,*} \right) \\
&= \left(\frac{\alpha_s}{2\pi} \right)^2 \mathcal{T}_{\tilde{g}g}^0(q^2) (q^2)^{-2\epsilon} \left[NN_F \left(\frac{2}{3\epsilon^3} + \frac{67}{18\epsilon^2} + \frac{1}{\epsilon} \left(\frac{326}{27} - \frac{8\pi^2}{9} \right) + \left(\frac{9215}{216} - \frac{275\pi^2}{72} - \frac{94}{9}\zeta_3 \right) \right) \right. \\
&\quad \left. + \frac{N_F}{N} \left(-\frac{1}{6\epsilon^3} - \frac{35}{36\epsilon^2} + \frac{1}{\epsilon} \left(-\frac{509}{108} + \frac{\pi^2}{4} \right) + \left(-\frac{1670}{81} + \frac{35\pi^2}{24} + \frac{31}{9}\zeta_3 \right) \right) \right. \\
&\quad \left. + N_F^2 \left(-\frac{1}{9\epsilon^2} + \left(\frac{91}{81} - \frac{\pi^2}{27} \right) \right) + \mathcal{O}(\epsilon) \right]. \quad (5.2)
\end{aligned}$$

Two different four-parton final states appear in the quark-gluon antenna functions at NNLO: $qggg$ and $q\bar{q}'\bar{q}'g$. The Lorentz and Dirac structure of these antenna functions is contained in the neutralino decay processes $\tilde{\chi} \rightarrow \tilde{g}ggg$ and $\tilde{\chi} \rightarrow \tilde{g}q\bar{q}g$. In contrast to the tree level three-parton neutralino decay matrix elements, which contained only one non-trivial colour ordering each, these four-parton matrix elements both contain several colour-orderings.

To expose the colour-ordered subamplitudes contributing to $\tilde{\chi}(q) \rightarrow \tilde{g}(p_1)g(p_3)g(p_4)g(p_5)$, we again contract the amplitude with $\sqrt{2}T_{i_1 i_2}^{a_1}$. The amplitude can then be expressed as sum over the permutations of the gluon colour indices:

$$\mathcal{M}_{\tilde{g}_1 g_3 g_4 g_5}^0 = i\eta g^4 \frac{1}{\sqrt{2}} \sum_{(i,j,k) \in P_C(3,4,5)} \left[(T^{a_i} T^{a_j} T^{a_k})_{i_1 i_2} - \frac{1}{N} \delta_{i_1 i_2} \text{Tr}(T^{a_i} T^{a_j} T^{a_k}) \right] M_{\tilde{g}ggg}^0(p_1, p_i, p_j, p_k), \quad (5.3)$$

where the sum runs only over cyclic permutations, since the colour-ordered amplitudes $M_{\tilde{g}_1 ggg}^0$ each contain the difference of two colour-orderings which are inverse to each other, as shown in Figure 2. It can be shown that the $1/N$ -term in the above expression does not contribute to the physical scattering amplitude [20].

The resulting squared matrix element, averaged over identical final state gluon permutations is

$$\frac{1}{3!} |\mathcal{M}_{\tilde{g}_1 g_3 g_4 g_5}^0|^2 = \eta^2 g^4 \frac{N^2 - 1}{16} \frac{1}{3!} N^2 \sum_{(i,j,k) \in P_C(3,4,5)} |M_{\tilde{g}ggg}^0(p_1, p_i, p_j, p_k)|^2. \quad (5.4)$$

It should be noted that this squared matrix element contains only the leading colour term obtained from the squares of the individual colour-ordered amplitudes, as expected in the colour ordered formulation for a process with three gluons [19, 20].

The tree level amplitude for $\tilde{\chi}(q) \rightarrow \tilde{g}(p_1)q(p_3)\bar{q}(p_4)g(p_5)$, contracted with $\sqrt{2}T_{i_1 i_2}^{a_1}$ contains three colour structures,

$$\begin{aligned}
\mathcal{M}_{\tilde{g}_1 q_3 \bar{q}_4 g_5}^0 &= i\eta g^2 (-i\sqrt{2}) \left[T_{i_1 i_4}^{a_5} \delta_{i_3 i_2} M_{\tilde{g}q\bar{q}g}^0(p_1, p_3, p_4, p_5) + T_{i_3 i_2}^{a_5} \delta_{i_1 i_4} M_{\tilde{g}q\bar{q}g}^0(p_1, p_4, p_3, p_5) \right. \\
&\quad \left. - \frac{1}{N} T_{i_3 i_4}^{a_5} \delta_{i_1 i_2} \tilde{M}_{\tilde{g}q\bar{q}g}^0(p_1, p_4, p_3, p_5) \right]. \quad (5.5)
\end{aligned}$$

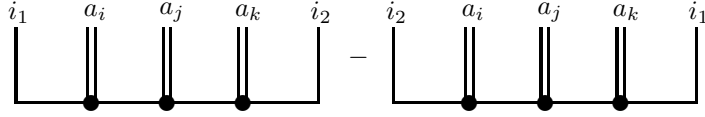


Figure 2: Colour flow contained in the colour ordered amplitude $M_{\tilde{g}ggg}^0(p_1, p_i, p_j, p_k)$ contributing to the tree level decay $\tilde{\chi} \rightarrow \tilde{g}ggg$.



Figure 3: Colour flow contained in the colour ordered amplitudes $M_{\tilde{g}q\bar{q}g}^0(p_1, p_3, p_4, p_5)$ (left) and $\tilde{M}_{\tilde{g}q\bar{q}g}^0(p_1, p_3, p_4, p_5)$ (right) contributing to the tree level decay $\tilde{\chi} \rightarrow \tilde{g}q\bar{q}g$.

The relation between leading and subleading colour ordered amplitudes is

$$M_{\tilde{g}q\bar{q}g}^0(p_1, p_3, p_4, p_5) + M_{\tilde{g}q\bar{q}g}^0(p_1, p_4, p_3, p_5) = \tilde{M}_{\tilde{g}q\bar{q}g}^0(p_1, p_3, p_4, p_5). \quad (5.6)$$

The squared matrix element reads

$$|\mathcal{M}_{\tilde{g}_1 q_3 \bar{q}_4 g_5}^0|^2 = \eta^2 g^4 (N^2 - 1) N_F \left\{ N \left[|M_{\tilde{g}q\bar{q}g}^0(p_1, p_3, p_4, p_5)|^2 + |M_{\tilde{g}q\bar{q}g}^0(p_1, p_4, p_3, p_5)|^2 \right] - \frac{1}{N} |\tilde{M}_{\tilde{g}q\bar{q}g}^0(p_1, p_3, p_4, p_5)|^2 \right\}. \quad (5.7)$$

It can be seen that this neutralino decay matrix element contains the same colour-ordered antenna structures, displayed in Figure 3, as the five-parton matrix element $\gamma^* \rightarrow q\bar{q}q'\bar{q}'g$ [9], relevant to $e^+e^- \rightarrow 3j$ at NNLO: gluon (p_5) emission between the colour-connected pairs (p_1, p_3) and (p_1, p_4) at leading colour, and gluon emission inside the (p_3, p_4) pair at subleading colour. In the latter case, the (p_3, p_5, p_4) system forms a colour singlet, such that the gluon p_5 acts as a photon and p_1 becomes a photino which decouples completely from any singular limit.

The four-parton tree-level neutralino matrix elements can be integrated over the tripole phase space [13], thus making their infrared singularity structure explicit,

$$\begin{aligned} \mathcal{T}_{\tilde{g}ggg}^2(q^2) &\equiv \int d\Phi_{T, \tilde{g}ggg} \frac{1}{3!} |\mathcal{M}_{\tilde{g}_1 g_3 g_4 g_5}^0|^2 \\ &= \left(\frac{\alpha_s}{2\pi}\right)^2 \mathcal{T}_{\tilde{g}g}^0(q^2) (q^2)^{-2\epsilon} N^2 \left[\frac{5}{2\epsilon^4} + \frac{37}{4\epsilon^3} + \frac{1}{\epsilon^2} \left(\frac{398}{9} - \frac{11\pi^2}{3} \right) \right. \\ &\quad \left. + \frac{1}{\epsilon} \left(\frac{28319}{144} - \frac{55\pi^2}{4} - \frac{188}{3}\zeta_3 \right) + \left(\frac{2201527}{2592} - \frac{529\pi^2}{8} - \frac{722}{3}\zeta_3 + \frac{511\pi^4}{720} \right) + \mathcal{O}(\epsilon) \right], \quad (5.8) \\ \mathcal{T}_{\tilde{g}q\bar{q}g}^2(q^2) &\equiv \int d\Phi_{T, \tilde{g}q\bar{q}g} |\mathcal{M}_{\tilde{g}_1 q_3 \bar{q}_4 g_5}^0|^2 \\ &= \left(\frac{\alpha_s}{2\pi}\right)^2 \mathcal{T}_{\tilde{g}g}^0(q^2) (q^2)^{-2\epsilon} \left[N N_F \left(-\frac{5}{6\epsilon^3} - \frac{17}{4\epsilon^2} + \frac{1}{\epsilon} \left(-\frac{2239}{108} + \frac{5\pi^2}{4} \right) \right. \right. \\ &\quad \left. \left. + \left(-\frac{20521}{216} + \frac{51\pi^2}{8} + \frac{200}{9}\zeta_3 \right) \right) \right] \end{aligned}$$

$$+\frac{N_F}{N}\left(\frac{1}{6\epsilon^3}+\frac{35}{36\epsilon^2}+\frac{1}{\epsilon}\left(\frac{1045}{216}-\frac{\pi^2}{4}\right)+\left(\frac{28637}{1296}-\frac{35\pi^2}{24}-\frac{40}{9}\zeta_3\right)\right)+\mathcal{O}(\epsilon)\Big]. \quad (5.9)$$

The sum of all NNLO subtraction terms yields the following infrared pole structure, which can be expressed in terms of NNLO infrared singularity operators [25],

$$\begin{aligned} & \mathcal{Poles} \left(\mathcal{T}_{\bar{g}g\bar{g}}^2(q^2) + \mathcal{T}_{\bar{g}q\bar{q}}^2(q^2) + \mathcal{T}_{\bar{g}ggg}^2(q^2) + \mathcal{T}_{\bar{g}q\bar{q}g}^2(q^2) \right) \\ &= \left(\frac{\alpha_s}{2\pi} \right)^2 \mathcal{T}_{\bar{g}g}^0(q^2) (q^2)^{-2\epsilon} \left[N^2 \left(-\frac{2}{\epsilon^4} - \frac{113}{12\epsilon^3} + \frac{1}{\epsilon^2} \left(-\frac{27}{4} + \frac{9\pi^2}{4} \right) + \frac{1}{\epsilon} \left(\frac{1049}{432} + \frac{11\pi^2}{3} + \frac{28}{3}\zeta_3 \right) \right) \right. \\ & \quad \left. + NN_F \left(\frac{7}{6\epsilon^3} + \frac{61}{36\epsilon^2} + \frac{1}{\epsilon} \left(-\frac{55}{54} - \frac{5\pi^2}{12} \right) \right) + \frac{N_F}{N} \left(\frac{1}{8\epsilon} \right) + N_F^2 \left(-\frac{1}{9\epsilon^2} \right) + \mathcal{O}(\epsilon^0) \right] \\ &= - \left(\frac{\alpha_s}{2\pi} \right)^2 \text{Re} \left[-2\mathbf{I}_{qg}^{(1)}(\epsilon, q^2) \left(2\mathbf{I}_{qg}^{(1)}(\epsilon, q^2) + 2\mathbf{I}_{qg}^{(1),*}(\epsilon, q^2) \right) \mathcal{T}_{\bar{g}g}^0(q^2) - 2\frac{\beta_0}{\epsilon} 2\mathbf{I}_{qg}^{(1)}(\epsilon, q^2) \mathcal{T}_{\bar{g}g}^0(q^2) \right. \\ & \quad + 4\mathbf{I}_{qg}^{(1)}(\epsilon, q^2) \mathcal{T}_{\bar{g}g}^1(q^2) + 2e^{-\epsilon\gamma} \frac{\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)} \left(\frac{\beta_0}{\epsilon} + K \right) 2\mathbf{I}_{qg}^{(1)}(2\epsilon, q^2) \mathcal{T}_{\bar{g}g}^0(q^2) \\ & \quad \left. + 2\mathbf{H}_{\bar{g}g}^{(2)}(\epsilon, q^2) \mathcal{T}_{\bar{g}g}^0(q^2) \right], \quad (5.10) \end{aligned}$$

where β_0 is the first term of the QCD β -function (2.5) and the constant K

$$K = \left(\frac{67}{18} - \frac{\pi^2}{6} \right) N - \frac{5}{9} N_F. \quad (5.12)$$

This structure coincides precisely with the singularity structure predicted in [25] for the purely virtual (two-loop times tree plus one-loop self-interference) NNLO corrections to a tree level process containing *two* quark-gluon antenna functions. The final state dependent constant $\mathbf{H}_{\bar{g}g}^{(2)}(\epsilon, q^2)$ contributes only at $\mathcal{O}(\epsilon^{-1})$:

$$\mathbf{H}_{\bar{g}g}^{(2)}(\epsilon, q^2) = \frac{e^{\epsilon\gamma}}{4\epsilon\Gamma(1-\epsilon)} \left(H_g^{(2)} + H_{\bar{g}}^{(2)} \right) (-q^2)^{-2\epsilon}. \quad (5.13)$$

It can be related to the known constants determining the ϵ^{-1} poles of four-parton two-loop matrix elements involving i quarks and j gluons [3]:

$$\mathbf{H}_{iq,jg}^{(2)}(\epsilon, q^2) = \frac{e^{\epsilon\gamma}}{4\epsilon\Gamma(1-\epsilon)} \left(iH_q^{(2)} + jH_g^{(2)} \right) (-q^2)^{-2\epsilon} \quad (5.14)$$

with

$$\begin{aligned} H_g^{(2)} &= \left(\frac{1}{2}\zeta_3 + \frac{5}{12} + \frac{11\pi^2}{144} \right) N^2 + \frac{5}{27} N_F^2 + \left(-\frac{\pi^2}{72} - \frac{89}{108} \right) NN_F - \frac{N_F}{4N}, \\ H_q^{(2)} &= \frac{N^2-1}{N^2} \left[\left(\frac{1}{4}\zeta_3 + \frac{41}{108} + \frac{\pi^2}{96} \right) N^2 + \left(\frac{3}{2}\zeta_3 + \frac{3}{32} - \frac{\pi^2}{8} \right) (N^2+1) + \left(\frac{\pi^2}{48} - \frac{25}{216} \right) NN_F \right], \\ H_{\bar{g}}^{(2)} &= \left(\frac{1}{4}\zeta_3 + \frac{41}{108} + \frac{\pi^2}{96} \right) (2N^2) + \left(\frac{\pi^2}{48} - \frac{25}{216} \right) \frac{2(N^2-1)N_F}{N} - \left(\frac{13}{14} - \frac{\pi^2}{8} + \frac{1}{2}\zeta_3 \right) (2N^2). \quad (5.15) \end{aligned}$$

In these equations, we decomposed the $H_i^{(2)}$ according to their colour structures. The coefficient (N^2+1) in front of the subleading colour contribution to $H_q^{(2)}$ arises due to the fact that abelian diagrams contributing to $qggg\bar{q}$ final states carry this colour structure, such that the generic (planar) leading colour contribution is given by just the first term in $H_q^{(2)}$ (see also Equation (3.6) of [9]).

It can be seen that $H_g^{(2)}$ contains twice the leading colour and the flavour dependent terms of $H_q^{(2)}$. The subleading colour term is absent, and the last term can be identified with the contribution to $H_q^{(2)}$ from singularities arising from final states containing two quark-antiquark pairs of identical flavour (Equation (4.51) of [16])¹.

Equations (5.11) and (5.15) demonstrate that the NNLO three and four parton contributions to neutralino decay into a gluino and massless partons display the same singularity structure as final state observables containing adjacent quark-gluon pairs, provided that the colour factors are adjusted correctly. It is therefore possible to construct colour-ordered quark-gluon antenna functions from the neutralino decay matrix elements derived here from the effective Lagrangian density (2.1).

6 Conclusions and Outlook

QCD antenna functions describe the behaviour of QCD matrix elements in their infrared singular limits, corresponding to soft or collinear parton emission. They are constructed so that they describe all singular limits arising from emission of unresolved partons in between the two colour-connected hard partons that define the antenna. The quark-antiquark antenna function is directly related to the physical matrix elements for $\gamma^* \rightarrow q\bar{q}$ +gluons. However, up to now, the NLO quark-gluon and gluon-gluon antenna functions [6] were constructed by starting from the quark-antiquark antenna function and adding terms to match the remaining limits contained in them. It does not appear feasible to extend this procedure to higher orders.

In this paper, we demonstrated that quark-gluon QCD antenna functions to all orders can be derived from an effective Lagrangian describing the decay of a massive neutralino into a massless gluino and the gluon field. In the colour ordered formalism underlying the antenna functions, the Majorana nature of the gluino allows it to appear simultaneously as quark and as antiquark. We demonstrated that the physical neutralino decay matrix elements reproduce the singular structure of QCD quark-gluon antenna functions at NLO and NNLO. We extracted the infrared structure for decay kinematics, as required for jet observables without partons in the initial state. By analytic continuation, the matrix elements derived here can also be continued to production (leading order process contains partons only in the initial state) or scattering (leading order process contains partons in initial and final state) kinematics, where they have to be integrated over the appropriate phase spaces.

All QCD antenna functions can be derived (as opposed to constructed) from physical matrix elements: quark-antiquark antennae from the decay of a virtual photon into partons, quark-gluon antennae from neutralino decay into gluino plus partons and finally gluon-gluon antennae [26] from Higgs boson decay into partons through the effective Lagrangian [27] coupling the Higgs field to the gluonic field strength tensor. The NNLO antenna subtraction functions obtained through this procedure will be reported in a subsequent publication [28].

Acknowledgements

The authors would like to thank Werner Porod and Daniel Wyler for numerous discussions on neutralino interactions.

This research was supported in part by the Swiss National Science Foundation (SNF) under contracts PMPD2-106101 and 200021-101874, by the UK Particle Physics and Astronomy Research Council and by the EU Fifth Framework Programme ‘Improving Human Potential’, Research Training Network ‘Particle Physics Phenomenology at High Energy Colliders’, contract HPRN-CT-2000-00149.

References

- [1] R.K. Ellis, W.J. Stirling and B.R. Webber, *QCD and Collider Physics*, Cambridge University Press (Cambridge, 1996);

¹In neutralino decay, this identical-flavour contribution corresponds to final states with three gluinos and one gluon. Being a SUSY QCD correction, it is discarded here, and has to be subtracted from $H_g^{(2)}$.

- G. Dissertori, I.G. Knowles and M. Schmelling, *Quantum Chromodynamics: High Energy Experiments and Theory*, Oxford University Press (Oxford, 2003).
- [2] S. Moch, J.A.M. Vermaseren and A. Vogt, Nucl. Phys. B **688** (2004) 101 [hep-ph/0403192];
A. Vogt, S. Moch and J.A.M. Vermaseren, Nucl. Phys. B **691** (2004) 129 [hep-ph/0404111].
- [3] Z. Bern, L.J. Dixon and A. Ghinculov, Phys. Rev. D **63** (2001) 053007 [hep-ph/0010075];
C. Anastasiou, E.W.N. Glover, C. Oleari and M.E. Tejeda-Yeomans, Nucl. Phys. B **601** (2001) 318 [hep-ph/0010212]; **601** (2001) 347 [hep-ph/0011094]; **605** (2001) 486 [hep-ph/0101304];
E.W.N. Glover, C. Oleari and M.E. Tejeda-Yeomans, Nucl. Phys. **605** (2001) 467 [hep-ph/0102201];
C. Anastasiou, E.W.N. Glover and M.E. Tejeda-Yeomans, Nucl. Phys. B **629** (2002) 255 [hep-ph/0201274];
E.W.N. Glover and M.E. Tejeda-Yeomans, JHEP **0306** (2003) 033 [hep-ph/0304169];
E.W.N. Glover, JHEP **0404** (2004) 021 [hep-ph/0401119];
Z. Bern, A. De Freitas and L.J. Dixon, JHEP **0109** (2001) 037 [hep-ph/0109078]; JHEP **0203** (2002) 018 [hep-ph/0201161]; JHEP **0306** (2003) 028 [hep-ph/0304168];
Z. Bern, A. De Freitas, L.J. Dixon, A. Ghinculov and H.L. Wong, JHEP **0111** (2001) 031 [hep-ph/0109079];
T. Binoth, E.W.N. Glover, P. Marquard and J.J. van der Bij, JHEP **0205** (2002) 060 [hep-ph/0202266];
L.W. Garland, T. Gehrmann, E.W.N. Glover, A. Koukoutsakis and E. Remiddi, Nucl. Phys. B **627** (2002) 107 [hep-ph/0112081] and **642** (2002) 227 [hep-ph/0206067];
S. Moch, P. Uwer and S. Weinzierl, Phys. Rev. D **66** (2002) 114001 [hep-ph/0207043];
T. Gehrmann and E. Remiddi, Nucl. Phys. B **640** (2002) 379 [hep-ph/0207020].
- [4] F.V. Tkachov, Phys. Lett. **100B** (1981) 65;
K.G. Chetyrkin and F.V. Tkachov, Nucl. Phys. **B192** (1981) 159;
V.A. Smirnov, Phys. Lett. **B460** (1999) 397 [hep-ph/9905323];
J.B. Tausk, Phys. Lett. **B469** (1999) 225 [hep-ph/9909506];
T. Binoth and G. Heinrich, Nucl. Phys. B **585** (2000) 741 [hep-ph/0004013];
S. Laporta, Int. J. Mod. Phys. A **15** (2000) 5087 [hep-ph/0102033];
T. Gehrmann and E. Remiddi, Nucl. Phys. B **580** (2000) 485 [hep-ph/9912329];
S. Moch, P. Uwer and S. Weinzierl, J. Math. Phys. **43** (2002) 3363 [hep-ph/0110083];
S. Weinzierl, Comput. Phys. Commun. **145** (2002) 357 [math-ph/0201011];
C. Anastasiou and K. Melnikov, Nucl. Phys. B **646** (2002) 220 [hep-ph/0207004];
V.A. Smirnov, *Evaluating Feynman Integrals*, Springer Tracts of Modern Physics (Heidelberg, 2004).
- [5] L.J. Dixon and A. Signer, Phys. Rev. Lett. **78** (1997) 811 [hep-ph/9609460]; Phys. Rev. D **56** (1997) 4031 [hep-ph/9706285];
Z. Nagy and Z. Trocsanyi, Phys. Rev. Lett. **79** (1997) 3604 [hep-ph/9707309];
S. Weinzierl and D.A. Kosower, Phys. Rev. D **60** (1999) 054028 [hep-ph/9901277];
W.B. Kilgore and W.T. Giele, Phys. Rev. D **55** (1997) 7183 [hep-ph/9610433];
Z. Nagy and Z. Trocsanyi, Phys. Rev. Lett. **87** (2001) 082001 [hep-ph/0104315];
Z. Nagy, Phys. Rev. Lett. **88** (2002) 122003 [hep-ph/0110315]; Phys. Rev. D **68** (2003) 094002 [hep-ph/0307268];
J. Campbell and R.K. Ellis, Phys. Rev. D **65** (2002) 113007 [hep-ph/0202176].
- [6] J. Campbell, M.A. Cullen and E.W.N. Glover, Eur. Phys. J. C **9** (1999) 245 [hep-ph/9809429].
- [7] Z. Bern, L.J. Dixon, D.C. Dunbar and D.A. Kosower, Nucl. Phys. B **425** (1994) 217 [hep-ph/9403226];
D.A. Kosower, Nucl. Phys. B **552** (1999) 319 [hep-ph/9901201];
D.A. Kosower and P. Uwer, Nucl. Phys. B **563** (1999) 477 [hep-ph/9903515];
Z. Bern, V. Del Duca and C.R. Schmidt, Phys. Lett. B **445** (1998) 168 [hep-ph/9810409];
Z. Bern, V. Del Duca, W.B. Kilgore and C.R. Schmidt, Phys. Rev. D **60** (1999) 116001 [hep-ph/9903516];
S. Catani and M. Grazzini, Nucl. Phys. B **591** (2000) 435 [hep-ph/0007142];
D.A. Kosower, Phys. Rev. Lett. **91** (2003) 061602 [hep-ph/0301069];
S. Weinzierl, JHEP **0307** (2003) 052 [hep-ph/0306248].

- [8] Z. Bern, L.J. Dixon and D.A. Kosower, Phys. Rev. Lett. **70** (1993) 2677 [hep-ph/9302280];
Z. Kunszt, A. Signer and Z. Trocsanyi, Phys. Lett. B **336** (1994) 529 [hep-ph/9405386];
Z. Bern, L.J. Dixon and D.A. Kosower, Nucl. Phys. B **437** (1995) 259 [hep-ph/9409393];
E.W.N. Glover and D.J. Miller, Phys. Lett. B **396** (1997) 257 [hep-ph/9609474];
Z. Bern, L.J. Dixon, D.A. Kosower and S. Weinzierl, Nucl. Phys. B **489** (1997) 3 [hep-ph/9610370];
J.M. Campbell, E.W.N. Glover and D.J. Miller, Phys. Lett. B **409** (1997) 503 [hep-ph/9706297];
Z. Bern, L.J. Dixon and D.A. Kosower, Nucl. Phys. B **513** (1998) 3 [hep-ph/9708239].
- [9] A. Gehrmann-De Ridder and E.W.N. Glover, Nucl. Phys. B **517** (1998) 269 [hep-ph/9707224];
J. Campbell and E.W.N. Glover, Nucl. Phys. B **527** (1998) 264 [hep-ph/9710255].
- [10] S. Catani and M. Grazzini, Phys. Lett. B **446** (1999) 143 [hep-ph/9810389]; Nucl. Phys. B **570** (2000) 287 [hep-ph/9908523];
F.A. Berends and W.T. Giele, Nucl. Phys. B **313** (1989) 595;
V. Del Duca, A. Frizzo and F. Maltoni, Nucl. Phys. B **568** (2000) 211 [hep-ph/9909464].
- [11] D.A. Kosower, Phys. Rev. D **67** (2003) 116003 [hep-ph/0212097];
S. Weinzierl, JHEP **0303** (2003) 062 [hep-ph/0302180];
W. B. Kilgore, Phys. Rev. D **70** (2004) 031501 [hep-ph/0403128];
M. Grazzini and S. Frixione, hep-ph/0411399.
- [12] G. Heinrich, Nucl. Phys. Proc. Suppl. **116** (2003) 368 [hep-ph/0211144];
C. Anastasiou, K. Melnikov and F. Petriello, Phys. Rev. D **69** (2004) 076010 [hep-ph/0311311];
T. Binoth and G. Heinrich, Nucl. Phys. B **693** (2004) 134 [hep-ph/0402265];
G. Heinrich, Nucl. Phys. Proc. Suppl. **135** (2004) 290 [hep-ph/0406332].
- [13] A. Gehrmann-De Ridder, T. Gehrmann and G. Heinrich, Nucl. Phys. B **682** (2004) 265 [hep-ph/0311276].
- [14] C. Anastasiou, K. Melnikov and F. Petriello, Phys. Rev. Lett. **93** (2004) 032002 [hep-ph/0402280].
- [15] C. Anastasiou, K. Melnikov and F. Petriello, hep-ph/0409088, hep-ph/0501130.
- [16] A. Gehrmann-De Ridder, T. Gehrmann and E.W.N. Glover, Nucl. Phys. B **691** (2004) 195 [hep-ph/0403057].
- [17] D.A. Kosower, Phys. Rev. D **57** (1998) 5410 [hep-ph/9710213].
- [18] A. Gehrmann-De Ridder, T. Gehrmann and E.W.N. Glover, Nucl. Phys. Proc. Suppl. **135** (2004) 97 [hep-ph/0407023].
- [19] F.A. Berends and W.T. Giele, Nucl. Phys. B **294** (1987) 700;
M. Mangano, S. Parke and Z. Xu, Nucl. Phys. B **298** (1988) 653;
M. Mangano, Nucl. Phys. B **309** (1988) 461;
L.J. Dixon, Proceedings of “Theoretical Advanced Study Institute in Elementary Particle Physics (TASI ’95): QCD and Beyond”, ed. D. Soper, World Scientific (Singapore, 1996), p.539 [hep-ph/9601359].
- [20] V. Del Duca, L.J. Dixon and F. Maltoni, Nucl. Phys. B **571** (2000) 51 [hep-ph/9910563];
F. Maltoni, K. Paul, T. Stelzer and S. Willenbrock, Phys. Rev. D **67** (2003) 014026 [hep-ph/0209271].
- [21] H.P. Nilles, Phys. Rept. **110** (1984) 1;
H.E. Haber and G.L. Kane, Phys. Rept. **117** (1985) 75.
- [22] H.E. Haber and D. Wyler, Nucl. Phys. B **323**, 267 (1989).
- [23] H. Baer, X. Tata and J. Woodside, Phys. Rev. D **42** (1990) 1568.
- [24] J. Rosiek, Phys. Rev. D **41** (1990) 3464; hep-ph/9511250.

- [25] S. Catani, Phys. Lett. B **427** (1998) 161 [hep-ph/9802439];
G. Sterman and M.E. Tejeda-Yeomans, Phys. Lett. B **552** (2003) 48 [hep-ph/0210130].
- [26] A. Gehrmann-De Ridder, T. Gehrmann and E.W.N. Glover, hep-ph/0502110.
- [27] F. Wilczek, Phys. Rev. Lett. **39** (1977) 1304;
M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Phys. Lett. B **78** (1978) 443.
- [28] A. Gehrmann-De Ridder, T. Gehrmann and E.W.N. Glover, in preparation.